Modeling Opinion Dynamics Using the Affine Boomerang Model

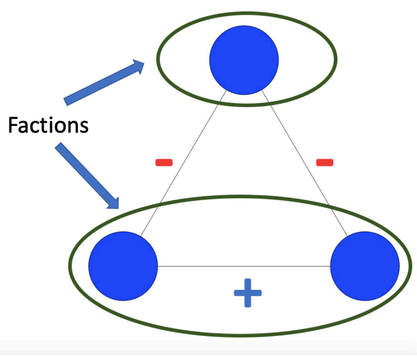
Emily E. Lopez, Elizabeth Y. Huang, Pedro Cisneros-Velarde, Francesco Bullo
Center for Control, Dynamical Systems and Computation
University of California at Santa Barbara

Introduction and Motivation

People's opinions are greatly influenced by those they interact with, which can be analyzed using social network analysis, the study of interpersonal behavior. Social network analysis can provide a stronger understanding of the dynamics of opinions with its use of models, having applications in marketing, political science, and information dissemination. The aim of this study is to use the discrete Affine Boomerang Model to observe how friendly and unfriendly relationships between individuals drives a group's opinion. Particularly, we focus on determining the social structures that promote polarization, the state when individuals either become strongly for or strongly against a social matter, and what factors influence the rate a group polarizes.

Affine Boomerang Model

Social groups are represented by graphs.



Legend

- Nodes: individuals
- Edges: Relationships
  - (+) = Friendly
  - (-) = Unfriendly

A **faction** is formed when a group of individuals have a friendly relationship.

Model Variables

Each individual *i* is assumed to have:

- $x_i(0)$ = **initial opinion** for a statement being discussed, denoted by $x_i(0) \in [0,1]$
  - $x_i(0) = 0$ if individual *i* completely disagrees with the statement being discussed
  - $x_i(0) = 1$ if individual *i* completely agrees with the statement being discussed
- a_i = **attachment to initial opinion** denoted by $a_i \in [0,1]$
  - $a_i = 0$ if individual *i is* openminded
  - $a_i = 1$ if individual *i* is closed-minded

At each time step t, connected individuals, *i* and *j*, are randomly selected from a uniform distribution. If *i* and *j* are friendly, they will update their opinion to come closer to agreement. If they dislike each other, their opinions will be updated to grow farther apart, coming to increasing disagreement. Mathematically, this is represented as

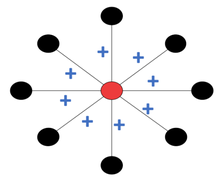
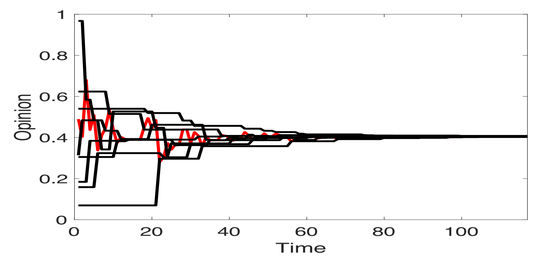
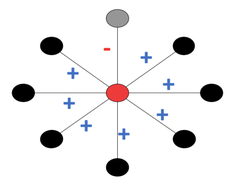
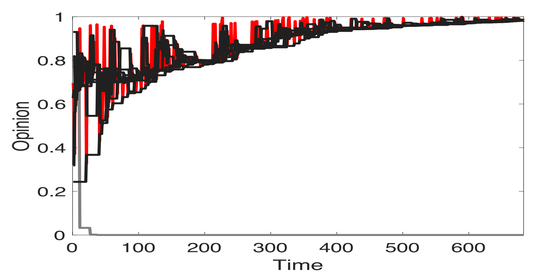
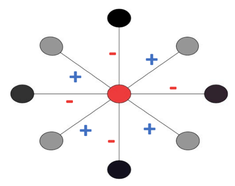
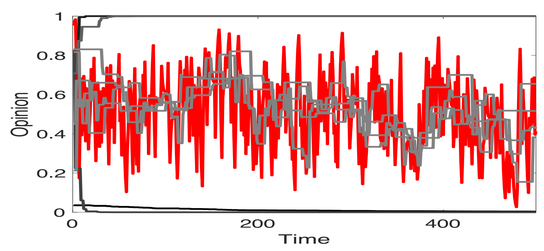
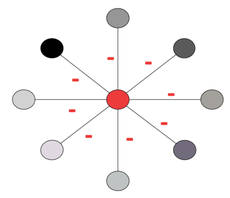
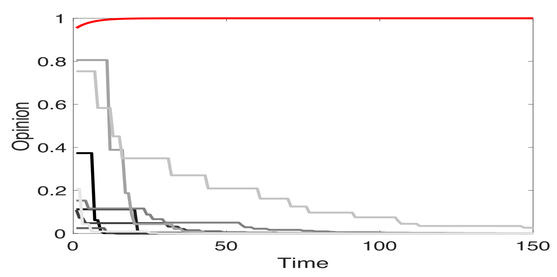
$$x_i(t+1) = \begin{cases} a_i x_i(t) + (1-a_i)x_j(t) & \text{if } \{i,j\} \text{ is a friendly relationship} \\ a_i x_i(t) + (1-a_i)(0) & \text{if } \{i,j\} \text{ is an unfriendly relationship and } x_i(t) < x_j(t) \\ a_i x_i(t) + (1-a_i)(1) & \text{if } \{i,j\} \text{ is an unfriendly relationship and } x_i(t) \geq x_j(t) \end{cases}$$

and similarly for individual j.

Results

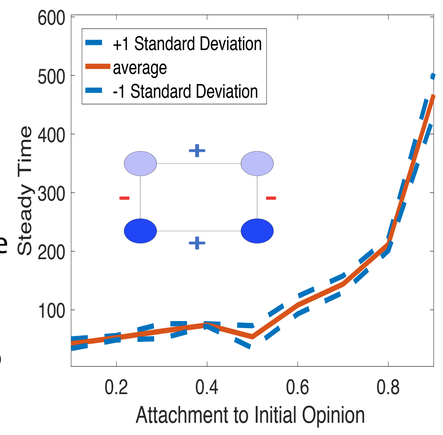
Question 1: Which conditions promote polarization?

We apply the Affine Boomerang Model on a 9-node star graph with the initial opinions randomized and fix all individuals to be moderate minded (a_i =0.5). Each graph that is a shade of gray reflects the behavior of a faction and the red graph denotes the behavior of the central node.

Faction Number and Behavior	Simulation
1 faction comes to consensus 	
2 factions polarize 	
3-8 factions may polarize or may not reach a steady opinion 	
9 factions polarize 	

Question 2: What affects polarization time?

We fix every node to have the same degree of attachment to their opinion, starting at 0.1. For each attachment value, we run the simulation on a cycle with two factions 100 times and find the average time it takes to reach complete polarization. We see that as people's attachment increases, the time it takes to reach complete polarization increases.



Conclusion

It can be concluded that the number of factions can influence whether individuals in a social group in the form of a star graph comes to agreement, polarizes, or cannot come to a stable opinion. Furthermore, the more individuals are attached to their opinions, the longer it takes for the group to polarize assuming the individuals are in a social group of the form of a cycle with two factions. We hope to rigorously prove our observations and apply them to more complex social networks.

Sources

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Supported by the Early Undergraduate Research Experience for Knowledge Acquisition (EUREKA) Program.