Quantum Computing for Classical Public-Key Cryptosystems

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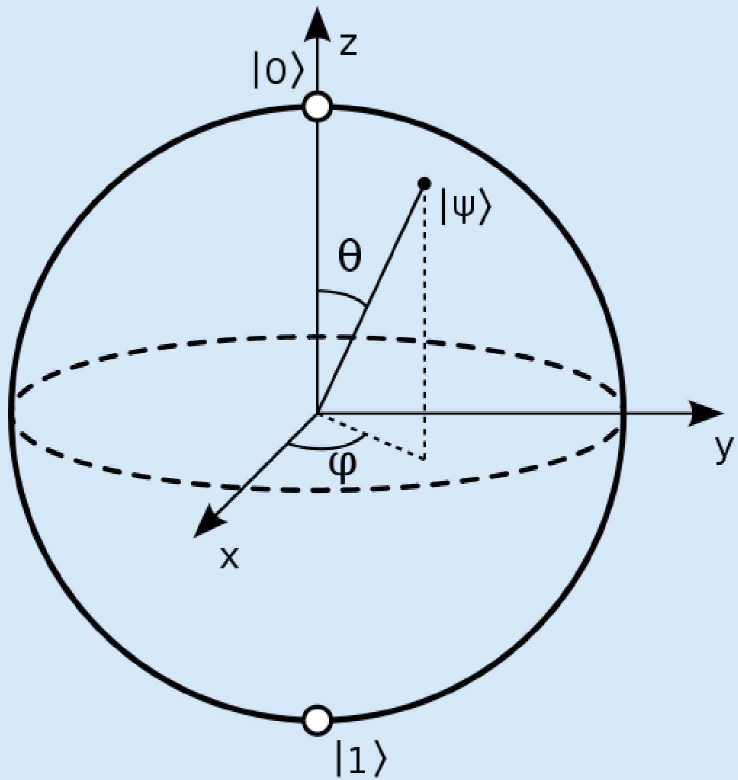
EUREKA Summer Internship
8/22/11

Quantum Computing



- Qubits vs. Bits
- "Quantum parallelism"
- Unitary transformations (rotations) only

Public-Key Cryptography

Security relies on hard mathematical problems defined by the public key which become easy when you know the private key

Example of Public-Key Cryptosystems:
RSA is based on the problem of factoring

Subset-Sum Problem

Given a set B of integers, what subset (if any) sums to an arbitrary integer S?

Example: { 319, 196, 250, 477, 200, 559 }, with the target S = 1605

Solution: 319 + 250 + 477 + 559 = 1605

Examples of cryptosystems based on the subset-sum problem:

Merkle-Hellman, Chor-Rivest

Is the cryptosystem secure?

1. Study public-key cryptosystems based on the Knapsack/Subset-Sum problem
2. Develop quantum algorithms to break such cryptosystems

Understanding Systems

Chor-Rivest Knapsack System
Benny Chor and Ronald L. Rivest
(1984)

→

Solved by
Serge Vaudenay
(1998)

Powerline System
Hendrik W. Lenstra Jr.
(1991)

→

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Quick Finite Field Intro

FF(p^h) = { $a_0 + a_1x + a_2x^2 + \dots + a_{h-1}x^{h-1}$ }, where a_i are in **FF**(p), so if p is an integer, { 1, 2, 3, … , p-1 }

Multiplicative generator: an element g such that g^n produces every element of the finite field except the zero element.

Chor-Rivest Knapsack

Public Key:

- **FF**(p^h), where p is prime and $h \leq p$
- $\{c_0, \ldots, c_{p-1}\}$, where $c_i = \log_g(x+i) + d$

Private Key:

- random multiplicative generator g of **FF**(p^h)
- random integer $0 \leq \mathbf{d} \leq p^h-2$

Chor-Rivest Knapsack

Encryption:

For message *m* with weight *h*, $\mathbf{E(m) = \Sigma m_i c_i}$

Decryption:

Factor the expression:

$$g^s \bmod f(x) + f(x),$$

where $\mathbf{s = E(m) - hd}$

The roots of the linear factors contain the message.
Factoring polynomials is easy.

Powerline System

Public Key:

- **FF**(q) and **FF**(q^h), where $q = p^n$
- Random set $S = \{ 1, 2, 3, \dots , s \}$ where $s \le q$
- $\{c_1, \dots, c_s\}$, $c_i = (ux-u\pi(i))^k$ of **FF**(q^h)

Private Key:

- Random element $u \in$ **FF**(q^h)
- random integer $1 \le k \le q^h-1$, where $\gcd(k, q^h-1) = 1$
- Map $\pi: S \rightarrow$ **FF**(q)

Powerline System

Encryption:

For message m with weight h, $\mathbf{E(m) = \prod c_i^{m_i}}$ **of FF(q^h)**

Decryption:

Factor the expression:

$$\mathbf{E(m)^l u^{-h} + f(x)},$$
$$\text{where } \mathbf{kl = 1 \bmod (q^h-1)}$$

The roots of the linear factors contain the message.
Factoring polynomials is easy.

Conclusion

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Discrete logarithm problem

Acknowledgments

Thank you ...
Professor Wim van Dam,
CNSI and EUREKA,
and Arica Lubin.

Made possible in part by
an NSF Career grant

Questions?